

Top-quark production at hadron colliders

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The current theoretical predictions for the observables related to the top-quark pair and the single-top productions at hadron colliders are briefly reviewed. The theoretical predictions are compared to the experimental measurements carried out at the Tevatron and the LHC.

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1. Introduction

The top quark is the heaviest elementary particle produced so far at colliders. Due to its large mass, it is expected to interact strongly with the electroweak symmetry breaking sector of the Standard Model (SM). Consequently, the top-quark plays a key role in the investigation of the origin of particle masses, both in the SM and in models of “new physics” (NP).

At hadron colliders, top quarks are produced via two production mechanisms: *i*) in “pairs”, $pp(\bar{p}) \rightarrow t\bar{t}$ and *ii*) as “single tops”, together with a bottom-quark jet, $pp(\bar{p}) \rightarrow t\bar{b}$, or a light-quark jet, $pp(\bar{p}) \rightarrow tq(\bar{q})$, or in association with a W boson, $pp(\bar{p}) \rightarrow tW$. The pair production process occurs more than twice as often as the single top production. Moreover, its experimental signature is cleaner. For these reasons, the top quark was originally discovered in $t\bar{t}$ events and it took 14 more years to detect single-top production events at the Tevatron.

The top quark has a very short lifetime: it decays almost exclusively in a b quark and a W boson in $\sim 5 \cdot 10^{-25}$ s. Since the top-quark lifetime is about one order of magnitude smaller than the hadronization time, the top is the only quark which does not hadronize. Consequently, the top-quark quantum numbers are accessible to the experimental measurements. Its spin and the exact nature of its couplings to the W boson can be studied starting from the geometrical distribution of the decay products. The top-quark mass, which together with the W mass plays an important role in constraining the Higgs mass via radiative corrections, can be measured with great accuracy, provided that a satisfactory theoretical definition of this parameter is employed.

The top-pair and the single-top production processes at hadron colliders can also be regarded as background for NP processes. In this short review the top quark events are considered as signal.

In the following we briefly review the status of the measurements of top-quark observables at the Tevatron and at the LHC, and we summarize the corresponding theoretical predictions.

2. Top-Quark at the Tevatron

From the top-quark discovery in 1995 [1] and until the shutdown on September 30, 2011, the top-quark production and decay have been extensively studied at the Fermilab Tevatron.

The channels used at Tevatron for the study of the pair production are three: the “di-lepton” channel, in which the two W s, which originate from the t and \bar{t} , decay leptonically; the “lepton+jets” channel, in which one of the W s decays leptonically while the other decays hadronically, and the “all-jets” channel, in which both W s decay hadronically. In the first case the experimental signature consists of a pair of high- p_T leptons, at least two jets (of which two originate from bottom quarks), and missing energy due to the neutrinos. In the “lepton+jets” channel, one finds one isolated high- p_T lepton, at least four jets (of which two are b jets), and missing energy. Finally in the “all-jets” channel the experimental signature consists of at least six jets, two of which are b jets, and no missing energy. The background processes to $t\bar{t}$ events are: W +jets, di-boson production, “all QCD” events, and Drell-Yan events. Roughly speaking, the background contains no b jets. Therefore, a very important tool to enhance the signal-to-background ratio is b tagging.

The single top production events are characterized by the so-called t -channel, s -channel, and tW associated production mechanisms. For all of them, events in which the W , produced by the top, decays leptonically are usually employed for the process detection. The experimental signature for

the t -channel consists of one isolated high- p_T lepton, with a b jet, a light-quark jet at high rapidity and missing energy. The s -channel signature consists in one high- p_T lepton, two b jets, and missing energy. The associated production cannot be seen at Tevatron. The background processes for the single top production coincide with the ones for the $t\bar{t}$ production and, in addition, the $t\bar{t}$ production itself is part of the background for the single top process.

The first top-related observable measured at the Tevatron was the total pair-production cross section, defined as $\sigma_{t\bar{t}} = (N - N_{bkg})/(\epsilon L)$, where N is the number of measured events, N_{bkg} is the number of background events, simulated by a MC event generator, ϵ is the pair selection efficiency (also simulated with a MC), and L is the luminosity, measured with data-driven techniques. A recent measurement of $\sigma_{t\bar{t}}$ by the CDF collaboration gives $\sigma_{t\bar{t}}^{(CDF)} = 7.5 \pm 0.48$ pb [2]. The experimental error corresponds to a relative error of $\Delta\sigma_{t\bar{t}}/\sigma_{t\bar{t}} \sim 6.5\%$. The measured pair production cross section is in good agreement with the SM value (see Section 4). The differential distribution of the top pair production cross section with respect to the $t\bar{t}$ invariant mass is interesting since it can reveal the presence of NP resonances in the spectrum. The measurement performed by the CDF collaboration is presented in [3]. The distribution is in agreement with the SM prediction [4]. The p_T distribution was measured by the D0 collaboration [5] and it also shows a good agreement with the corresponding SM result [6].

For what concerns the single top production, the current combined Tevatron measurement of the $(s+t)$ -channel cross section is $\sigma_{s+t} = 2.76_{-0.47}^{+0.58}$ pb [7], which is in good agreement with the SM value. This measurement also allows the determination of $|V_{tb}|$. However, the CDF collaboration registers a tension in the ratio of the s - and t -channel cross sections. The measured ratio is more than 2σ away from the SM prediction [8]. Finally, recently the cross section for the process $p\bar{p} \rightarrow t\bar{t} + \gamma$ was measured by the CDF collaboration [9], finding $\sigma_{t\bar{t}\gamma} = 0.18 \pm 0.08$ pb, in agreement with the SM prediction.

The current Tevatron combination of top-quark mass measurements is $m_t = 173.2 \pm 0.9$ GeV, with a relative error of only $\sim 0.5\%$ [10]. However, given the fact that the measurement is carried out by comparing data with MC simulations, and since the top-mass parameter used in the MC is not well defined theoretically, it would be desirable to have a measurement of m_t related to a well defined Lagrangian parameter. Recently, the D0 collaboration evaluated the on-shell and $\overline{\text{MS}}$ top masses by comparing data to the most up-to-dated theoretical predictions for the production cross section, finding a mean value for the on-shell m_t which is slightly below the value of 173.2 GeV, but still compatible with it within one standard deviation [11]. The width of the top quark is also measured at Tevatron. The limit reported by CDF is $\Gamma_t < 7.6$ GeV at the 95% CL, and the value measured by D0 is $\Gamma_t = 1.99_{-0.55}^{+0.69}$ GeV [12]. The difference between the top and the anti-top masses was measured by the D0 collaboration and it is compatible with zero: $\Delta m_t = 0.8 \pm 1.8(stat) \pm 0.5(syst)$ GeV [13].

The W helicity fractions, F_0 , F_R and F_L , are measured by fitting the one-parameter distribution of the positive charged lepton coming from the W decay. Using CDF and D0 measurements that simultaneously determine F_0 and F_R , one finds $F_0 = 0.732 \pm 0.081$, $F_R = -0.039 \pm 0.045$ [14], in full agreement with the NNLO SM predictions [15]. Also the spin correlations are in agreement with the SM predictions [16]. D0 measures a $t\bar{t}$ spin correlation strength, using as spin quantization axis the direction of the beam, of $C = 0.1_{-0.45}^{+0.45}$, while CDF finds $C = 0.72 \pm 0.64(stat) \pm 0.26(syst)$ [17]. However, the error bands are too big for any claim.

Tevatron experiments are also searching for NP in top-quark pair and single-top production processes. This activity includes searches for new resonances, as for instance a Z' or a W' , for possible anomalous couplings of the top quark to the W that can reveal a discrepancy with respect to the $V - A$ structure of the SM, the search for a fourth generation (with decays in SM particles) or for non-SM decays of the top quark, as for instance $t \rightarrow H^+ b \rightarrow q\bar{q}' b (\tau \nu b)$. However, so far, no evidence of NP was found, and the good agreement with the SM predictions is used to set constraints on the NP parameters, such as the masses and the couplings of the conjectured NP particles.

The only observable which exhibits a sizable discrepancy with respect to the corresponding SM prediction is the top-pair forward-backward asymmetry, A_{FB} . This observable is defined as $A_{FB}^{(i)} = (N_t(y_t > 0) - N_t(y_t < 0)) / (N_t(y_t > 0) + N_t(y_t < 0))$, where $N_t(y_t > 0)$ ($N_t(y_t < 0)$) is the number of top quarks with positive (negative) rapidity, and i labels the frame of reference in which the measurement of the rapidity is carried out. The asymmetry is measured either in the laboratory frame or in the $t\bar{t}$ rest frame. Assuming CP invariance (the SM CP violation is irrelevant here), the forward-backward asymmetry coincides with the charge asymmetry $A_C^{(i)} = (N_t(y_t > 0) - N_{\bar{t}}(y_{\bar{t}} > 0)) / (N_t(y_t > 0) + N_{\bar{t}}(y_{\bar{t}} > 0))$, since we have $N_{\bar{t}}(y_{\bar{t}} > 0) = N_t(y_t < 0)$. The measurement of the forward backward asymmetry in the $t\bar{t}$ rest frame can be carried out by employing the fact that the difference between the top and anti-top rapidities $\Delta y = y_t - y_{\bar{t}}$ is invariant with respects to boosts along the beam axis, and that it is by definition equal to twice the top-quark rapidity in the $t\bar{t}$ frame: $\Delta y = 2y_t^{\bar{t}}$. Consequently, one find that $A_{FB}^{(i)} = (N_t(\Delta y > 0) - N_t(\Delta y < 0)) / (N_t(\Delta y > 0) + N_t(\Delta y < 0))$. The measurement of the asymmetry is obtained by employing “lepton+jets” events. The charge of the observed lepton allows one to determine whether the parent parton is a top or an anti-top, and therefore to know also the nature of the other top in the pair. The latter gives origin to the W boson which decays hadronically. The momentum of the top quarks which decays hadronically can be reconstructed, since all of the jets originating from the decay are detected. CDF collaboration also uses “di-leptonic” events. In this case the analysis is complicated by the missing energy due to the neutrinos. CDF and D0 collaborations find comparable forward-backward asymmetry values, $A_{FB}^{(t\bar{t}),CDF} = 0.201 \pm 0.067$ and $A_{FB}^{(t\bar{t}),D0} = 0.196 \pm 0.065$ [18], which are more than 2σ larger than the SM theoretical predictions at the NLO [19]. The asymmetry value in the laboratory frame measured by CDF is $A_{FB}^{(lab),CDF} = 0.150 \pm 0.055$ [20]. Moreover, CDF registers a further discrepancy with the SM value in the dependence of $A_{FB}^{(t\bar{t})}$ with respect both to the $t\bar{t}$ invariant mass and rapidity difference. While for $m_{t\bar{t}} < 450\text{GeV}$ the CDF result is compatible with the SM prediction within one standard deviation, for $m_{t\bar{t}} > 450\text{GeV}$ the measured value, $A_{FB}^{(t\bar{t})}(m_{t\bar{t}} > 450\text{GeV}) = 0.475 \pm 0.114$ is 3.4σ bigger [20] than the SM prediction. The same happens for the dependence on the rapidity difference. At small Δy , we find $A_{FB}^{(t\bar{t})}(|\Delta y| \leq 1.0) = 0.026 \pm 0.104 \pm 0.055$ compatible with the SM prediction. At large rapidity difference, CDF finds $A_{FB}^{(t\bar{t})}(|\Delta y| \geq 1.0) = 0.611 \pm 0.210 \pm 0.141$ [21], which is much larger than the SM prediction $A_{FB}^{(t\bar{t})}(|\Delta y| \geq 1.0) = 0.123 \pm 0.018$ [22]. However, neither the increase of the asymmetry for large values of the pair invariant mass, nor the increase of the asymmetry for large values of $|\Delta y|$ is at the moment confirmed by D0 [18]. For a dedicated review on possible NP explanations of the A_{FB} discrepancy we refer the reader to [23].

3. Top-Quark at the LHC

Since the end of 2010, also CMS and ATLAS collaborations at the LHC are producing accurate measurements of the top properties. The most recent values are based on $\sim 1 \text{ fb}^{-1}$ of data, recorded between the end of 2010 and Summer 2011.

The $t\bar{t}$ production cross section was measured by both collaborations: the measured values are $\sigma_{t\bar{t}}^{(CMS)} = 158 \pm 19 \text{ pb}$ and $\sigma_{t\bar{t}}^{(ATLAS)} = 176 \pm 5(\text{stat})_{-10}^{+13}(\text{syst}) \pm 7(\text{lum}) \text{ pb}$ [24]. Therefore, after only few months of data taking, the relative error on this observable is already quite small ($\sim 10 - 15\%$); furthermore the statistical uncertainty is already smaller than the systematic one. The t -channel single top production cross section is measured with a larger relative error of $\sim 30\%$: $\sigma_t^{(CMS)} = 83.6 \pm 29.8(\text{stat} + \text{syst}) \pm 3.3(\text{lum}) \text{ pb}$ and $\sigma_t^{(ATLAS)} = 90_{-22}^{+32} \text{ pb}$ [25].

The top-quark mass was measured by the ATLAS collaboration using a template method (which suffers of the same problems already pointed out in the previous section): the value obtained is $m_t^{(ATLAS)} = 175.9 \pm 0.9(\text{stat.}) \pm 2.7(\text{syst.}) \text{ GeV}$ [26]. The CMS collaboration repeated the analysis done by D0, using the theoretical cross section and measuring the on-shell and $\overline{\text{MS}}$ top masses [27], finding comparable results. The difference between the top and anti-top masses was measured by CMS, which obtained a value compatible with zero: $\Delta m_t = -1.2 \pm 1.2(\text{stat.}) \pm 0.5(\text{syst.}) \text{ GeV}$ [28].

W -helicity fractions and spin correlations, measured by ATLAS, are also compatible with their SM value: $F_0 = 0.75 \pm 0.08$, $F_L = 0.25 \pm 0.08$ (setting $F_R = 0$) [29], and $\kappa = 0.34_{-0.11}^{+0.15}$ in the helicity base [30].

Finally, although LHC is a machine with a symmetric initial state and therefore the A_{FB} measured at the Tevatron cannot be seen, one can define and measure a charge asymmetry by exploiting the fact that the rapidity distributions of top and anti-top quarks are different. The antitops tend to be produced at small rapidity, while the distribution of the tops is broader. The relevant observable is defined as $A_C = (N(\Delta|y| > 0) - N(\Delta|y| < 0)) / (N(\Delta|y| > 0) + N(\Delta|y| < 0))$, with $\Delta|y| = |y_t| - |y_{\bar{t}}|$ and y_t is the top rapidity; the measured values for this quantity are $A_C^{(CMS)} = -0.013 \pm 0.026(\text{stat})_{-0.021}^{+0.026}(\text{syst})$ [31] and $A_C^{(ATLAS)} = -0.024 \pm 0.016(\text{stat}) \pm 0.023(\text{syst})$ [32]. Due to the large errors, both values are compatible with the SM prediction, which is $\sim +1\%$.

The already remarkable accuracy of the LHC measurements is going to improve in the next years. For example, in the high-luminosity and high-energy phase, the $t\bar{t}$ production cross section is expected to be measured with an accuracy of 5%, while the single-top t -channel cross section will be measured with an accuracy of 10%. These very precise experimental measurements must be matched by equally accurate theoretical predictions.

4. Theoretical Predictions

The production of a top-antitop pairs is dominated by the strong interaction. The inclusive production cross section can be written using the QCD Factorization Theorem as

$$\sigma_{h_1, h_2}^{t\bar{t}}(s_{\text{had}}, m_t^2) = \sum_{ij} \int_{4m_t^2}^{s_{\text{had}}} d\hat{s} L_{ij}(\hat{s}, s_{\text{had}}, \mu_f^2) \hat{\sigma}_{ij}(\hat{s}, m_t^2, \mu_f^2, \mu_r^2), \quad (4.1)$$

where the hard scattering of the partons i and j ($i, j \in \{q, \bar{q}, g\}$) at a partonic center of mass energy \hat{s} is described by the partonic cross section, $\hat{\sigma}_{ij}$, which can be calculated in perturbative QCD. The

process independent partonic luminosity, L_{ij} , describes the probability of finding, in the hadrons h_1 and h_2 (where $h_1, h_2 = p, \bar{p}$ at the Tevatron, while $h_1, h_2 = p, p$ at the LHC), an initial state involving partons i and j with the given partonic energy \hat{s} . The integration extends up to the collider hadronic c. m. energy s_{had} . μ_f and μ_r indicate the renormalization and factorization scales.

At leading order (LO) in perturbation theory, there are two partonic channels contributing to the $t\bar{t}$ production cross section: the quark-antiquark channel $q\bar{q} \rightarrow t\bar{t}$ and the gluon fusion channel $gg \rightarrow t\bar{t}$. Because of the interplay between parton luminosity and partonic cross sections, the quark-antiquark channel dominates the pair production cross section at the Tevatron, while at the LHC the inclusive cross section is largely dominated by gluon fusion events.

In single-top production, there are three LO partonic channels: *i*) $q(\bar{q})b \rightarrow q'(\bar{q}')t$, in which a W boson is exchanged in the t channel, *ii*) $q\bar{q}' \rightarrow t\bar{b}$, in which the W boson is exchanged in the s channel, and *iii*) the “associated tW production” $gb \rightarrow tW$. The t -channel process dominates the single top production both at the Tevatron and at the LHC. The s -channel production was detected at the Tevatron and it plays no role at the LHC. The associated production, instead, cannot be revealed at the Tevatron while is the second most important single-top production mechanism at the LHC.

4.1 NLO Calculations

The LO predictions for the $t\bar{t}$ or single-top production cross sections are affected by a huge dependence on the renormalization/factorization scales, and they cannot be regarded as reliable predictions. More accurate predictions can be obtained by taking into account the NLO corrections, which consist of two parts: the virtual corrections, originating from the interference of the one-loop diagrams with the tree-level ones, and the real radiation corrections, originating from the interference of the $2 \rightarrow 3$ amplitudes. For totally inclusive quantities, as for instance the total cross section, one has to integrate the final-state particles over the complete phase space. This is the approach used for instance in [33]. In so doing, the IR divergences of the virtual part cancel exactly (analytically) against the divergences which originate from the integration of the additional parton in particular regions of the phase space. However, in order to compare directly the theoretical predictions with the experimental measurements, one needs to impose cuts and to take into account the geometrical acceptance of the detectors. Consequently, for more exclusive observables a subtraction scheme is needed to “regularize” the IR collinear and soft divergences, coming from the integration over the phase space. The basic idea is the following. The NLO cross section for the production of n partons in the final state is given by the sum of the virtual part, integrated over the n -particle phase space, and the real part, integrated over the $n+1$ -particle phase space: $\sigma_{NLO} = \int_n \sigma_V + \int_{n+1} \sigma_R$. The UV divergences are removed by the renormalization procedure. However, the virtual part exhibits IR divergences, that appear as poles in $\epsilon = (4-D)/2$, where D is the dimension of the space-time. The same divergences, with opposite sign, arise after the integration of the real radiation over the phase space. In order to locally regularize the IR divergences, one adds (and subtracts) a term σ_S that reproduces the matrix element behavior in all singular limits and, at the same time, is sufficiently simple to be integrated analytically: $\sigma_{NLO} = \int_n (\sigma_V - \int_1 \sigma_S) + \int_{n+1} (\sigma_R + \sigma_S)$. The integration $\int_1 \sigma_S$ reproduces the poles of the virtual part, while the integration of the $n+1$ final state partons is now finite and it can be carried out numerically in 4 dimensions. At NLO, the subtraction terms are completely known and several subtraction formalisms are available in the literature [34].

4.1.1 Stable Tops

Let us first consider the t and \bar{t} in the final state as stable on-shell particles. The NLO QCD corrections to the total $t\bar{t}$ cross section, summed over the final spins and colors, were calculated by many groups [35]. They enhance the cross section by almost 25% at the Tevatron and by 50% at the LHC. The residual renormalization/factorization scale dependence, plus parton distribution functions uncertainties, is about 15-20%. The EW corrections are also known [36], but their contribution (+1% at Tevatron and -0.5% at the LHC) is negligible in comparison to the QCD theoretical error.

The NLO QCD corrections to the t -channel single-top production are moderate. They enhance the cross section by 9% at the Tevatron and by 5% at the LHC [37, 38]. The NLO EW corrections decrease the cross section by 1% both at the Tevatron and the LHC [39]. The NLO QCD corrections to the s -channel cross section are large, resulting in an enhancement of 47% at the Tevatron and 44% at the LHC [38, 40]. Finally, the NLO QCD corrections to the associated tW production enhance the cross section by 10% at the LHC [41].

For what concerns processes with additional particles in the final state, the NLO corrections to $t\bar{t} + j$ were calculated in [42] (the calculated cross section at the Tevatron, $\sigma_{t\bar{t}j} = 1.79^{+0.16}_{-0.31}$ pb is in good agreement with the CDF measurement [43]) and those to $t\bar{t} + 2j$ in [44]. Moreover, the $t\bar{t}b\bar{b}$ production was considered in [45]. The NLO corrections to the production of a top pair in association with a photon were calculated in [46].

NLO corrections to many processes concerning both $t\bar{t}$ -pair and single-top productions are matched with parton showers in the publicly available codes MC@NLO [47] and POWHEG [48].

4.1.2 Resummation

The QCD corrections to processes that involve at least two large energy scales (the partonic energy $\sqrt{\hat{s}}$ and the top mass m_t are both much larger than Λ_{QCD}) are characterized by a logarithmic behavior in the vicinity of the boundary of the phase space: $\sigma \sim \sum_{m,n} C_{mn} \alpha_s^m \log^n(\rho)$, where ρ is the kinematic variable that “measures” the distance from the exclusive boundary. When $\rho \ll 1$, even if the transverse momentum is such that $\alpha_s(Q^2) \ll 1$ and perturbative QCD can be employed, one can find that $\alpha_s^m \log^n(\rho) \sim \mathcal{O}(1)$. The logarithmically enhanced terms spoil the convergence of the fixed-order expansion, that has to be recovered by performing a systematic resummation of these terms to all orders in perturbation theory [49]. For the $t\bar{t}$ pair production process, the resummation of the leading logarithmic terms (LL) was carried out in [50], and the next-to-leading logarithmic (NLL) in [51], both for the cross section at the production threshold and for the invariant mass distribution. Recently, the NNLL resummation was carried out for the cross section at threshold, the top-pair invariant mass distribution, the top-quark transverse momentum distribution, and top-quark rapidity distribution [52, 53, 4, 6]. Different approaches employing either Mellin space resummation or momentum space resummation techniques based on Soft Collinear Effective Theory were employed in these works. For the single top production, the NLL terms were calculated in [54] and the NNLL in [55]. A comprehensive review of the recent results obtained with resummation techniques can be found in [56].

4.1.3 Factorisable Corrections

In the papers reviewed in the last two sections, the top quarks are treated as stable particles.

However, only hadrons and leptons are detected experimentally and it is on these particles that the experimental cuts are imposed. Therefore, it is highly desirable to consider the actual final state in the theoretical analysis. This is very difficult from the point of view of the calculation, since one needs to deal with Feynman diagrams with many external legs. A first step towards this goal consists in working within the “narrow-width approximation”: since the top-quark behaves as a narrow resonance, *i.e.* $\Gamma_t/m_t \ll 1$, one can formally take the limit $\Gamma_t/m_t \rightarrow 0$ of the complete cross section. The limit $\Gamma_t/m_t \rightarrow 0$ decouples the top-quark production process from the top-quark decay. This approach allows one to compute realistic distributions and to keep trace of the spin of the tops. It was applied both to the top-pair and the single-top production processes.

Two groups performed detailed studies of the top-quark pair production within the narrow width approximation approach [16, 57, 58]. For single-top production, the same formalism was used to study the t -channel cross section at the LHC [59]. Recently, also the off-shell effects both for t - and s -channel cross sections at the Tevatron and at the LHC were computed [60].

4.1.4 Non-Factorisable Corrections for Pair Production

In 2010, two groups calculated the complete set of corrections to $pp(\bar{p}) \rightarrow W^+W^-b\bar{b}$, including also the non-factorisable corrections [61]. The calculation is extremely challenging and it involves almost 1500 Feynman diagrams with six external legs. As a by-product of the calculation, the authors could prove that for inclusive quantities the non-factorisable corrections are indeed of $\mathcal{O}(\Gamma_t/m_t) \sim 1\%$. With these results many exclusive observables, with realistic experimental cuts, can be evaluated.

4.2 Towards a NNLO Analysis of the Top-Pair Production in Perturbative QCD

The foreseen accuracy with which the LHC will be able to measure some of the top-pair production observables is such that in several cases the calculation of the NNLO corrections is required. While in the single-top production the NLO theoretical predictions (supplemented by the soft gluon resummation) already match the expected experimental accuracy, this is not the case for the $t\bar{t}$ production process. In the latter case the inclusion of the NNLO corrections in the analysis is needed. Due to the complexity of the calculations, to date the complete set of NNLO QCD corrections is not yet available. However, many partial results are known, and the full calculation of the NNLO correction appears to be within reach.

The most accurate theoretical predictions currently employed for comparison with the experimental measurements include the “approximate NNLO” corrections (NLO plus some or all of the following ingredients: scale dependence at NNLO, Coulomb terms up to two loops, logarithmic terms obtained by re-expanding NNLL formulas), both for $t\bar{t}$ [52] and for single top [62] productions.

Many parts of the full top-pair NNLO matrix element are known. In [63] the matrix elements for the $q\bar{q}$ and gg channels were computed in the $s \gg m_t^2$ limit. In [64], matrix elements in the $q\bar{q}$ channel were computed numerically and by retaining the full dependence on the top-quark mass. In [65], all of the IR two-loop poles, both in the $q\bar{q}$ and gg channels, were evaluated analytically. In [66], the virtual one-loop times one-loop matrix elements were calculated. Finally, in [67] the two-loop fermionic and leading color corrections to the $q\bar{q}$ channel and the two-loop leading color corrections to the gg channel were computed analytically by employing the Laporta algorithm [68]

(as implemented in the C++ code Reduze [69]), and the differential equation method [70]. While the most complicated four-point master integrals were evaluated especially for these projects (see [71]), part of the needed master integrals were already available in the literature [72].

The computation of exclusive observables at the NNLO requires a subtraction scheme for the real corrections in presence of massive partons. A complete NNLO subtraction scheme applicable to the top-pair production is not yet available. However, many intermediate results were recently obtained. The approach employed follows closely the one adopted at the NLO. In order to locally regularize the IR divergences that originate from the integration over the phase space, one adds and subtracts terms that reproduce the behavior of the matrix element in all the singular limits. At NNLO the structure of the singularities due to unresolved partons in the final state is more involved with respect to the NLO [73, 74]. One encounters double unresolved singularities and overlapping singularities. Furthermore, when integrating the subtraction terms, one needs to evaluate complicated two-loop integrals. Subtraction terms, together with their integrated counterpart, were published so far in different frameworks [75]. Numerical and semi-analytical techniques, based on sector decomposition, were also applied to this problem [76].

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